

# Newtonian limit of String-Dilaton Gravity

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February 7, 2008

## Abstract

We study the weak-field limit of string-dilaton gravity and derive corrections to the Newtonian potential which strength directly depends on the self interaction potential and the nonminimal coupling of the dilaton scalar field. We discuss also possible astrophysical applications of the results, in particular the flat rotation curves of spiral galaxies.

PACS number(s): 04.50.+h, 04.20. Cv, 98.80. Hw

Keyword(s): string-dilaton theory, Newtonian limit.

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# 1 Introduction

String-dilaton gravity seems to yield one of the most promising scenarios in order to solve several shortcomings of standard and inflationary cosmology [1]. First of all, it addresses the problem of initial singularity which is elegantly solved by invoking a maximal space-time curvature directly related to the string size [2].

Besides, it introduces a wide family of cosmological solutions which comes out thanks to the existence of the peculiar symmetry called *duality* which holds at string fundamental scales as well as at cosmological scales [3]. In practice, if  $a(t)$  is a cosmological solution of a string-dilaton model, also  $a^{-1}(t)$  has to be one by a time reversal  $t \rightarrow -t$ . In this case, one can study the evolution of the universe towards  $t \rightarrow +\infty$  as well as towards  $t \rightarrow -\infty$ . The junction of these two classes of solutions at some maximal value of curvature (considered as branches of more general solutions) eventually should be in agreement with inflationary paradigm and solve the initial value and singularity problems of standard cosmological model [2].

The main interest for string-dilaton cosmological models is related to the fact that they come from the low-energy limit of (super)string theory which can be considered one of the most serious attempt, in the last thirty years, to get the great unification. This theory avoids the shortcomings of quantum field theories due, essentially, to the point-like nature of particles (renormalization) and includes gravity in the same conceptual scheme of the other fundamental interactions (the graviton is just a string *mode* as the other gauge bosons [4]).

However, despite of theoretical results, we are very far from the possibility to test experimentally the full predictions of the theory. The main reason for this failure is that the Planck scales, where the string effects become relevant, are too far from the experimental capabilities of today high-energy physics. Cosmology remains the only open way to observational investigations since the today detectable *remnants* of primordial processes could be a test for the theory. Furthermore, a lot of open questions of astrophysics, as dark matter, relic gravitational wave background, large-scale structure, primordial magnetic fields and so on could be solved by strings and their dynamics (see, for example [2] and references therein).

The key element of string-dilaton gravity, in low-energy limit, is the fact that a dynamics, consistent with duality, can be implemented only by taking into account massless modes (zero modes) where the scalar mode (the dilaton) is nonminimally coupled to the other fields. The tree-level action, in general, contains a second-rank symmetric tensor field (the metric), a scalar field (the dilaton) and a second-rank antisymmetric tensor field (the so-called Kalb-Ramond *universal* axion). Such an action can be recast as a scalar-tensor theory, *e.g.* induced gravity, where the gravitational coupling is a function of the dilaton field [1, 5]. Then it is legitimate to study the Newtonian limit of the string-dilaton gravity to see what is its behaviour in the weak-field and slow-motion approximations. This approach is useful if we want to investigate how string-dilaton dynamics could affect shorter scales than cosmological ones. The issue is to search for

effects of the coupling and the self-interaction potential of dilaton also at scales of the order of Solar System or Galactic size.

This fact is matter of debate since several relativistic theories *do not* reproduce General Relativity's results but generalize them introducing corrections to the Newtonian potential which could have interesting physical consequences. For example, some theories give rise to terms capable of explaining the flat rotation curve of galaxies without using dark matter as the fourth-order conformal theory proposed by Mannheim *et al.* [6]. Others use Yukawa corrections to the Newton potential for the same purpose [7].

Besides, indications of an apparent, anomalous, long-range acceleration revealed from the data analysis of Pioneer 10/11, Galileo, and Ulysses spacecrafts could be framed in a general theoretical scheme by taking into account Yukawa-like or higher order corrections to the Newtonian potential [9].

In general, any relativistic theory of gravitation can yield corrections to the Newton potential which, in the post-Newtonian (PPN) formalism, could furnish tests for the same theory [10].

In this paper, we want to discuss the Newtonian limit of string-dilaton gravity. We develop our calculations in the string frame since we want to see what is the role of dilaton-nonminimal coupling in the recovering of Newtonian limit.

In Sec.2, we write down the string-dilaton field equations. The weak field approximation and the resolution of linearized equations are given in Sec.3. In Sec.4, we discuss the results specifying the possible astrophysical applications.

## 2 The string-dilaton field equations

The tree-level string-dilaton effective action, *i.e.* at the lowest order in loop expansion, containing all the massless modes, without higher-order curvature corrections of order  $\alpha'$  (*i.e.* without the Gauss-Bonnet invariant) is

$$\begin{aligned} \mathcal{A} = & -\frac{1}{2\lambda_s^{d-1}} \int d^{d+1}x \sqrt{-g} e^{-\phi} \left[ R + (\nabla\phi)^2 - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} + V(\phi) \right] + \\ & + \int d^{d+1}x \sqrt{-g} \mathcal{L}_m, \end{aligned} \quad (1)$$

where  $R$  is the Ricci scalar,  $\phi$  is the dilaton field,  $V(\phi)$  the dilaton self-interaction potential.  $H_{\mu\nu\alpha} = \partial_{[\mu} B_{\nu\alpha]}$  is the full antisymmetric derivative of the Kalb-Ramond axion tensor,  $\mathcal{L}_m$  is the Lagrangian density of other generic matter sources. The theory is formulated in  $d+1$ -dimensions and  $\lambda_s$  is the string fundamental minimal length parameter. The effective gravitational coupling, to lowest order, is given by

$$\sqrt{8\pi G_N} = \lambda_p = \lambda_s e^{\phi/2}, \quad (2)$$

where  $G_N$  is the Newton constant and  $\lambda_p$  is the Planck length. We choose units such that  $2\lambda_s^{d-1} = 1$  so that  $\exp \phi$  is the  $(d+1)$ -dimensional gravitational coupling. At the end of Sec.3, we will restore standard units.

The field equations are derived by varying the action (1) with respect to  $g_{\mu\nu}$ ,  $\phi$ , and  $B_{\mu\nu}$ . We get, respectively,

$$G_{\mu\nu} + \nabla_\mu \nabla_\nu \phi + \frac{1}{2} g_{\mu\nu} \left[ (\nabla\phi)^2 - 2\nabla^2\phi - V(\phi) + \frac{1}{12} H_{\alpha\beta\gamma} H^{\alpha\beta\gamma} \right] - \frac{1}{4} H_{\mu\alpha\beta} H_\nu{}^{\alpha\beta} = \quad (3)$$

$$= \frac{e^\phi}{2} T_{\mu\nu},$$

$$R + 2\nabla^2\phi - (\nabla\phi)^2 + V - V' - \frac{1}{12} H_{\mu\nu\alpha} H^{\mu\nu\alpha} = 0, \quad (4)$$

$$\nabla_\mu (e^{-\phi} H^{\mu\alpha\beta}) = 0, \quad (5)$$

where  $G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R$  is the Einstein tensor,  $T^{\mu\nu}$  is the stress-energy tensor of matter sources and  $V' = dV/d\phi$ .

We are assuming that standard matter is a perfect fluid minimally coupled to the dilaton. Otherwise, in the nonminimally coupled case, we should take into account a further term in Eq.(4).

The above ones are a system of tensor equations in  $d+1$  dimensions which assigns the dynamics of  $g_{\mu\nu}$ ,  $\phi$ , and  $B_{\mu\nu}$ . Now we take into account the weak field approximation in order to derive the PPN limit of the theory.

### 3 The weak field limit and the solution of linearized equations

As it is obvious, all the invariances of the full theory are not preserved if we linearize it. For example, we loose duality in the linearized solutions. However, this is not a problem in the present context since we are assuming a regime well far from early singularity where duality is adopted to solve cosmological shortcomings. Here, we want to investigate if *remnants* of primordial string-dilaton dynamics are detectable at our nearest scales (Solar System or Galaxy).

To recover the Newtonian limit, we write the metric tensor as

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad (6)$$

where  $\eta_{\mu\nu}$  is the Minkowski metric and  $h_{\mu\nu}$  is a small correction to it. In the same way, we define the scalar field  $\psi$  as a perturbation of the original field  $\phi$ , that is

$$\phi = \phi_0 + \psi, \quad (7)$$

where  $\phi_0$  is a constant. This assumption means that at scales where Newtonian limit holds, the effects of dilaton are small perturbations. However, as  $\psi \rightarrow 0$ , the standard gravitational coupling of General Relativity has to be restored. For the scalar potential,

we can assume a power law expression of the form  $V(\phi) = \lambda\phi^n$  so that, in the same limit of (7), we have

$$V(\phi) \simeq \lambda \left( \phi_0^n + n\phi_0^{n-1}\psi + \frac{n(n-1)}{2}\phi_0^{n-2}\psi^2 \right). \quad (8)$$

At this point, it is worthwhile to note that the parameters  $\lambda$  and  $n$  can be related to the number of spatial dimension  $d$  as it is shown in [11] for scalar-tensor theories of gravity. Below, we will show that suitable choices of  $\lambda$  and  $n$  give rise to interesting physical behaviours for the gravitational potential.

Finally, the weak field approximation for the axion gives only second order terms in the field equations with respect to  $h_{\mu\nu}$  and  $\psi$  so that we can discard its contribution in the following considerations. A physical interpretation of this fact could be that the production of primordial magnetic fields, considered as "seeds" for the today observed large magnetic fields of galaxies [12] is a second order effect if due to  $H_{\mu\nu\alpha}$ . This topic will be studied elsewhere.

Let us define now the auxiliary fields

$$\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h, \quad (9)$$

and

$$\sigma_\alpha \equiv \bar{h}_{\alpha\beta,\gamma}\eta^{\beta\gamma}. \quad (10)$$

The Einstein tensor  $G_{\mu\nu}$  becomes, at first order in  $h_{\mu\nu}$ ,

$$G_{\mu\nu} \simeq -\frac{1}{2} \left[ \square_\eta \bar{h}_{\mu\nu} + \eta_{\mu\nu} \sigma_{\alpha,\gamma}{}^\alpha - \sigma_{\mu,\nu} - \sigma_{\nu,\mu} \right], \quad (11)$$

where  $\square_\eta \equiv \eta^{\mu\nu}\partial_\mu\partial_\nu$ . We have not fixed the gauge yet.

Using the approximation (7), and the approximated expression of the scalar potential (8), the field equations (3) and (4) become, respectively,

$$\begin{aligned} \square_\eta \bar{h}_{\mu\nu} + \eta_{\mu\nu} \sigma_{\alpha,\gamma}{}^\alpha - (\sigma_{\mu,\nu} + \sigma_{\nu,\mu}) &= 2\psi_{,\mu\nu} + 2\eta_{\mu\nu} \square_\eta \psi + \\ &+ \lambda(\eta_{\mu\nu} + h_{\mu\nu})\phi_0^n + \lambda n\phi_0^{n-1}\eta_{\mu\nu}\psi \simeq -e^{\phi_0} T_{\mu\nu}, \end{aligned} \quad (12)$$

$$2\square_\eta \psi + \frac{1}{2}\square_\eta \bar{h} + \sigma_{\alpha,\gamma}{}^\alpha + \lambda\phi_0^n[n\phi_0^{-1} - n(n-1)\phi_0^{-2}]\psi + \lambda\phi_0^n[1 - n\phi_0^{-1}] = 0. \quad (13)$$

We have discarded the field equation (5) since it gives only higher than linear order terms.

We can eliminate the term proportional to  $\psi_{,\mu\nu}$  by choosing an appropriate gauge. In fact, by writing the auxiliary field  $\sigma_\alpha$  as

$$\sigma_\mu = -\psi_{,\mu}, \quad (14)$$

Eq.(12) reads

$$\square_\eta \bar{h}_{\mu\nu} + \eta_{\mu\nu} \square_\eta \psi + \lambda\phi_0^n \left[ \eta_{\mu\nu} \left( 1 + \frac{n}{\phi_0} \psi \right) + h_{\mu\nu} \right] \simeq -e^{\phi_0} T_{\mu\nu}. \quad (15)$$

In our approximations, we can neglect the terms in  $h_{\mu\nu}$  and  $\psi$  in Eq.(15) being  $h_{\mu\nu} \ll \eta_{\mu\nu}$  and  $\psi \ll 1$ . Eq.(15) becomes

$$\square_\eta \bar{h}_{\mu\nu} + \eta_{\mu\nu} \square_\eta \psi + \lambda \phi_0^n \eta_{\mu\nu} \simeq -e^{\phi_0} T_{\mu\nu}. \quad (16)$$

By defining the further auxiliary field

$$\tilde{h}_{\mu\nu} \equiv \bar{h}_{\mu\nu} + \eta_{\mu\nu} \psi, \quad (17)$$

we get the final form

$$\square_\eta \tilde{h}_{\mu\nu} + \lambda \phi_0^n \eta_{\mu\nu} \simeq -e^{-\phi_0} T_{\mu\nu}. \quad (18)$$

The original perturbation field  $h_{\mu\nu}$  can be written in terms of the new field as

$$h_{\mu\nu} = \tilde{h}_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} \tilde{h} + \eta_{\mu\nu} \psi, \quad (19)$$

being  $\tilde{h} \equiv \eta^{\mu\nu} \tilde{h}_{\mu\nu}$ .

We turn now to the Klein-Gordon Eq.(13). With a little algebra, it can be recast in the form

$$(\square_\eta + c_1^2) \psi \simeq -\frac{e^{-\phi_0}}{2} T - \Phi_0, \quad (20)$$

where  $T$  is the trace of the stress-energy tensor of standard matter and the constants are

$$c_1^2 = \lambda \phi_0^n [n(n-1)\phi_0^{-2} - n\phi_0^{-1}], \quad \Phi_0 = (3 - n\phi_0^{-1}) \lambda \phi_0^n. \quad (21)$$

We are working in the weak-field and slow motion limits, namely we assume that the matter stress-energy tensor  $T_{\mu\nu}$  is dominated by the mass density term. Furthermore, we neglect time derivatives with respect to the space derivatives, so that  $\square_\eta \rightarrow -\Delta$ , where  $\Delta$  is the ordinary Laplace operator in flat spacetime. The linearized field equations (18) and (20) have, for point-like distribution of matter<sup>1</sup> the following solutions

$$h_{00} \simeq -\frac{2G_N M}{r} (1 - e^{-c_1 r}) + c_2 r^2 + c_3 \cosh(c_1 r), \quad (22)$$

$$h_{ii} \simeq -\frac{2G_N M}{r} (1 + e^{-c_1 r}) - c_2 r^2 - c_3 \cosh(c_1 r), \quad (23)$$

$$\psi \simeq \frac{2G_N M}{r} e^{-c_1 r} + c_3 \cosh(c_1 r), \quad (24)$$

where

$$c_2 = 2\pi \lambda \phi_0^n, \quad c_3 = \frac{3 - n\phi_0^{-1}}{n(n-1)\phi_0^{-2} - n\phi_0^{-1}}. \quad (25)$$

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<sup>1</sup>To be precise, we can define a Schwarzschild mass of the form

$$M = \int (2T_0^0 - T_\mu^\mu) \sqrt{-g} d^3x,$$

and  $\rho(r) = M\delta(r)$ .

Here the functions (22) and (23) are the nonzero components of  $h_{\mu\nu}$  while Eq.(24) is the perturbation of the dilaton. We have restored standard units by using Eq.(2).

When  $\lambda = 0$  and in the slow motion limit, the  $(0, 0)$ -component of the field Eq.(18) reduces to the usual Poisson equation

$$\Delta\Phi = 4\pi G_N\rho. \quad (26)$$

Here  $\Phi$  is the Newtonian potential which is linked to the metric tensor by the relation  $h_{00} = 2\Phi$ .

What we have obtained are solutions of the linearized field equations, starting from the action of a scalar field nonminimally coupled to the geometry, and minimally coupled to the ordinary matter (specifically the string-dilaton gravity). Such solutions explicitly depend on the parameters  $\phi_0, n, \lambda$  which assign the model in the class (1).

## 4 Discussion and Conclusions

The above solution (22), as we said, can be read as a Newtonian potential with exponential and quadratic corrections *i.e.*

$$\Phi(r) \simeq -\frac{G_N M}{r}(1 - e^{-c_1 r}) + \frac{c_2}{2}r^2 + \frac{c_3}{2}\cosh(c_1 r). \quad (27)$$

In general, it can be shown [13],[15],[14] that most of the extended theories of gravity has a weak field limit of similar form, *i.e.*

$$\Phi(r) = -\frac{G_N M}{r} \left[ 1 + \sum_{k=1}^n \alpha_k e^{-r/r_k} \right], \quad (28)$$

where  $G_N$  is the value of the gravitational constant as measured at infinity,  $r_k$  is the interaction length of the  $k$ -th component of non-Newtonian corrections. The amplitude  $\alpha_k$  of each component is normalized to the standard Newtonian term; the sign of  $\alpha_k$  tells us if the corrections are attractive or repulsive (see [10] for further details). Besides, the variation of the gravitational coupling is involved. As an example, let us take into account only the first term of the series in (28) which is usually considered the leading term (this choice is not sufficient if other corrections are needed). We have

$$\Phi(r) = -\frac{G_N M}{r} \left[ 1 + \alpha_1 e^{-r/r_1} \right]. \quad (29)$$

The effect of non-Newtonian term can be parameterized by  $(\alpha_1, r_1)$ . For large distances, at which  $r \gg r_1$ , the exponential term vanishes and the gravitational coupling is  $G_N$ . If  $r \ll r_1$ , the exponential becomes unity and, by differentiating Eq.(29) and comparing with the gravitational force measured in laboratory, we get

$$G_{lab} = G_N \left[ 1 + \alpha_1 \left( 1 + \frac{r}{r_1} \right) e^{-r/r_1} \right] \simeq G_N(1 + \alpha_1), \quad (30)$$

where  $G_{lab} = 6.67 \times 10^{-8} \text{ g}^{-1} \text{ cm}^3 \text{ s}^{-2}$  is the usual Newton constant measured by Cavendish-like experiments. Of course,  $G_N$  and  $G_{lab}$  coincide in the standard gravity. It is worthwhile to note that, asymptotically, the inverse square law holds but the measured coupling constant differs by a factor  $(1 + \alpha_1)$ . In general, any correction introduces a characteristic length that acts at a certain scale for the self-gravitating systems. The range of  $r_k$  of the  $k$ th-component of non-Newtonian force can be identified with the mass  $m_k$  of a pseudo-particle whose Compton's length is

$$r_k = \frac{\hbar}{m_k c}. \quad (31)$$

The interpretation of this fact is that, in the weak energy limit, fundamental theories which attempt to unify gravity with the other forces introduce, in addition to the massless graviton, particles *with mass* which carry the gravitational force [16]. These masses introduce length scales which are

$$r_k = 2 \times 10^{-5} \left( \frac{1 \text{ eV}}{m_k} \right) \text{ cm}. \quad (32)$$

There have been several attempts to constrain  $r_k$  and  $\alpha_k$  (and then  $m_k$ ) by experiments on scales in the range  $1 \text{ cm} < r < 1000 \text{ km}$ , using totally different techniques [17],[18],[19]. The expected masses for particles which should carry the additional gravitational force are in the range  $10^{-13} \text{ eV} < m_k < 10^{-5} \text{ eV}$ . The general outcome of these experiments, even retaining only the term  $k = 1$ , is that a "geophysical window" between the laboratory and the astronomical scales has to be taken into account. In fact, the range

$$|\alpha_1| \sim 10^{-2}, \quad r_1 \sim 10^2 \div 10^3 \text{ m}, \quad (33)$$

is not excluded at all in this window. An interesting suggestion has been given by Fujii [20], which proposed that the exponential deviation from the Newtonian standard potential (the "fifth force") could arise from the microscopic interaction which couples to nuclear isospin and baryon number.

The astrophysical counterpart of these non-Newtonian corrections seemed ruled out till some years ago due to the fact that experimental tests of general relativity predict "exactly" the Newtonian potential in the weak energy limit, "inside" the Solar System. Recently, as we said above, indications of an anomalous, long-range acceleration revealed from the data analysis of Pioneer 10/11, Galileo, and Ulysses spacecrafts (which are now almost outside the Solar System) makes these Yukawa-like corrections come into play [9]. Besides, Sanders [7] reproduced the flat rotation curves of spiral galaxies by using

$$\alpha_1 = -0.92, \quad r_1 \sim 40 \text{ kpc}. \quad (34)$$

His main hypothesis is that the additional gravitational interaction is carried by an ultra-soft boson whose range of mass is  $m_1 \sim 10^{-27} \div 10^{-28} \text{ eV}$ . The action of this boson becomes efficient at galactic scales without the request of enormous amounts of dark matter to stabilize the systems.



Eckhardt [8] uses a combination of two exponential terms and gives a detailed explanation of the kinematics of galaxies and galaxy clusters, without dark matter models, using arguments similar to those of Sanders.

It is worthwhile to note that both the spacecrafts and galactic rotation curves indications are "outside" the usual Solar System boundaries used to test General Relativity. However, the above authors do not start from any fundamental theory in order to explain the outcome of Yukawa corrections. In their contexts, these terms are phenomenological.

Another important remark in this direction deserves the fact that some authors [21] interpret the recent experiments on cosmic microwave background as BOOMERANG [22] in the frame of *modified Newtonian dynamics* without invoking any dark matter model.

All these facts point towards the line of thinking that "corrections" to the standard gravity have to be seriously taken into account.

Let us turn now to the above solutions (22)–(24), in particular to the gravitational potential (27). This comes out from the weak field limit (PPN approximation) of a string-dilaton effective action (1). The specific model is singled out by the number of spatial dimension  $d$  and the form of self-interaction potential  $V(\phi)$ . We have considered the quite general class  $V(\phi) = \lambda\phi^n$ .

Without loosing of generality, we can assume  $\phi_0 = 1$  in Eq.(7). This means that for  $\phi = 1$  the standard gravitational coupling is restored in the action (1). However, the condition  $\psi \ll 1$  must hold in (7). For the choice  $n = 3$ , we have

$$\Phi(r) \simeq -\frac{G_N M}{r}(1 - e^{-c_1 r}) + \frac{c_2}{2}r^2. \quad (35)$$

where, beside the standard Newtonian potential, two corrections are present. Due to the definition of the constants  $c_{1,2}$  their strength directly depends on the coupling  $\lambda$  of the self-interaction potential  $V(\phi)$ . If  $\lambda > 0$ , from Eq.(21) and Eq.(25), we have that the first correction is a *repulsive* Yukawa-like term with  $\alpha_1 = -1$  and  $r_1 = c_1^{-1} = \lambda^{-1/2}$ . The second correction is given by a sort of positive-defined cosmological constant  $c_2$  which acts as a repulsive force<sup>2</sup> proportional to  $r$ .

If  $\lambda < 0$ , the first correction is oscillatory while the second is attractive.

From the astrophysical point of view, the first situation is more interesting. If we assume that the dilaton is an ultra-soft boson which carries the scalar mode of gravitational field, we get, by Eq.(32), that the length scale  $\sim 10^{22} \div 10^{23}$  cm, needed to explain the flat rotation curves of spiral galaxy, is obtained if its mass range is  $m \sim 10^{-27} \div 10^{-28}$  eV. The second correction to the Newtonian potential can contribute to stabilize the system being repulsive and acting as a constant density which is a sort of cosmological constant at galactic scales (see also [23] but the models which they used are different from our). In general, if  $\alpha_1 \sim -1$  the flat rotation curves of galaxies can be reconstructed [7].

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<sup>2</sup>We have to note that if in the Poisson equation we have a positive constant density, it gives rise to a repulsive quadratic potential in  $r$  and then to a linear force. A positive constant density can be easily interpreted as a sort of cosmological constant.

If the mass of the dilaton is in the range  $10^{-13}\text{eV} < m < 10^{-5}\text{eV}$  also the "geophysical windows" could be of interest. Finally the mass range  $m \sim 10^{-9} \div 10^{-10}\text{eV}$  could be interesting at Solar System scales (for the allowed mass windows in cosmology see [24]). Similar analysis can be performed also for other values of  $n$  which means other models of the class (1).

In conclusion, we have derived the weak energy limit of string-dilaton gravity showing that the Newtonian gravitational potential is corrected by exponential and quadratic terms. These terms introduce natural length scales which can be connected to the mass of the dilaton. If the dilaton is an ultra-soft boson, we can expect observable effects at astrophysical scales. If it is more massive, the effects could be interesting at geophysical or microscopic scales.

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